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Chaos, Solitons and Fractals 32 (2007) 937–943
CHAOS SOLITONS & FRACTALS
www.elsevier.com/locate/chaos

Synchronization of self-excited oscillators suspended on elastic structure

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Accepted 3 July 2006

Communicated by Professor G. Iovane

Abstract

We consider the dynamics of self-excited oscillators suspended on the elastic structure. We show that for the given conditions of the structure, initially uncorrelated oscillations of each oscillator can synchronize, i.e., all oscillators evolve periodically with the frequency of the oscillations of elastic structure. Basing on the theory of linear oscillations we introduced the mechanism responsible for the observed synchronization. We argue that the observed phenomena are generic in the parameter space and independent of the number of oscillators and their location on the elastic structure.

1. Introduction

The phenomenon of the synchronization of a number of self-excited oscillators by periodic signal is relatively well understood [1–11]. Such a synchronization can be easily detected by looking on whether the oscillations follow the excitation i.e., all oscillators oscillate with the same frequency or not. Recently, it has been shown that self-excited oscillators can synchronize under the influence of common chaotic signal (the so-called generalized synchronization) [12] or common random noise [13].

In the current studies we consider the synchronization of nonlinear self-excited oscillators located on (coupled through) elastic structure. In this case the oscillators are excited by the vibrations of the structure. The intensity of the excitation on a particular oscillator depends on the location of the oscillator on the elastic structure but all oscillators are excited by the signals with common frequency. We present a numerical study of a realistic model of two autonomous van der Pol oscillators suspended on the elastic beam.

We show that for the given conditions of the elastic structure initially uncorrelated oscillations of each oscillator can synchronize, i.e., all oscillators evolve periodically with the frequency of the oscillations of elastic beam. We found that two types of synchronization are possible: (i) oscillators are in phase and (ii) oscillators are in antiphase.

We argue that the observed phenomena are generic in the parameter space and independent of the number of oscillators and their location on the elastic structure. Additionally, we described the mechanism which explains the observed synchronization. The introduced mechanism is based on the theory of linear oscillations.

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0960-0779/$ - see front matter © 2006 Published by Elsevier Ltd.
doi:10.1016/j.chaos.2006.07.022

The paper is organized as follows. In Section 2 we recall some fundamental properties of van der Pol oscillator and introduce our model. Section 3 describes the observed phenomena of synchronization and introduces the mechanism responsible for it. Finally, our results are summarized in Section 4.

2. Beam–oscillators system

As an example of self-excited oscillator we considered van der Pol oscillator described by

$$m\ddot{y} + d_y(y^2 - 1)\dot{y} + k_y y = 0. \tag{1}$$

where m, d_y and k_y are constant. For negative d_y system (1) has a stable equilibrium ($y = \dot{y} = 0$). For positive d_y this equilibrium becomes unstable and the oscillator exhibits periodic self-excited oscillations – after initial time phase trajectories reach limit cycle attractor. Eq. (1) when excited by the periodic signal $Z\sin\omega t$, where Z and ω are respectively the amplitude and the frequency of the signal, show rich bifurcation behavior which ends with chaos [14–18].

We assumed that two different (in parameter values) van der Pol oscillators are connected to concentrated masses bases u_1 and u_2 and located on massless elastic beam as shown in Fig. 1. The evolution of the described four degree-of-freedom system is given by the following equation:

$$\begin{aligned}
&m_1\ddot{y}_1 + d_{y1}\left((y_1 - z_1)^2 - 1\right)(\dot{y}_1 - \dot{z}_1) + k_{y_1}(y_1 - z_1) = 0\\
&m_2\ddot{y}_2 + d_{y2}\left((y_2 - z_2)^2 - 1\right)(\dot{y}_2 - \dot{z}_2) + k_{y_2}(y_2 - z_2) = 0\\
&u_1\ddot{z}_1 + \vartheta u_1\dot{z}_1 + k_{11}z_1 + k_{12}z_2 - d_{y1}\left((y_1 - z_1)^2 - 1\right)(\dot{y}_1 - \dot{z}_1) - k_{y_1}(y_1 - z_1) = 0\\
&u_2\ddot{z}_2 + \vartheta u_2\dot{z}_2 + k_{21}z_1 + k_{22}z_2 - d_{y2}\left((y_2 - z_2)^2 - 1\right)(\dot{y}_2 - \dot{z}_2) - k_{y_2}(y_2 - z_2) = 0,
\end{aligned} \tag{2}$$

where k_{ij} is the matrix of stiffness coefficients and ϑ denotes the damping coefficient of the beam. The stiffness coefficients of the beam k_{ij} were calculated by the method based on beam deflection equation (Euler–Bernoulli law) $EId^2z/dx^2 = M_g$ [19], where E is the modulus of elasticity of the beam material, and I is the moment of inertia of the beam cross section about its central line, M_g is a bending moment. We employed the common linear model of the external damping with coefficients ϑu_1 and ϑu_2 proportional to values of concentrated masses u_1 and u_2.

3. Synchronization

In our numerical studies of the system (2) we took $l = 1.0$, $l_1 = 0.333$, $l_2 = 0.666$, $\vartheta = 0.2$, $u_1 = u_2 = 1.0$, $m_1 = m_2 = 1.0$, $d_{y1} = d_{y2} = 0.5$, $k_{y1} = 2.0$, $k_{y2} = 1.0$ and assumed beam stiffness EI as a control parameter. Fig. 2a

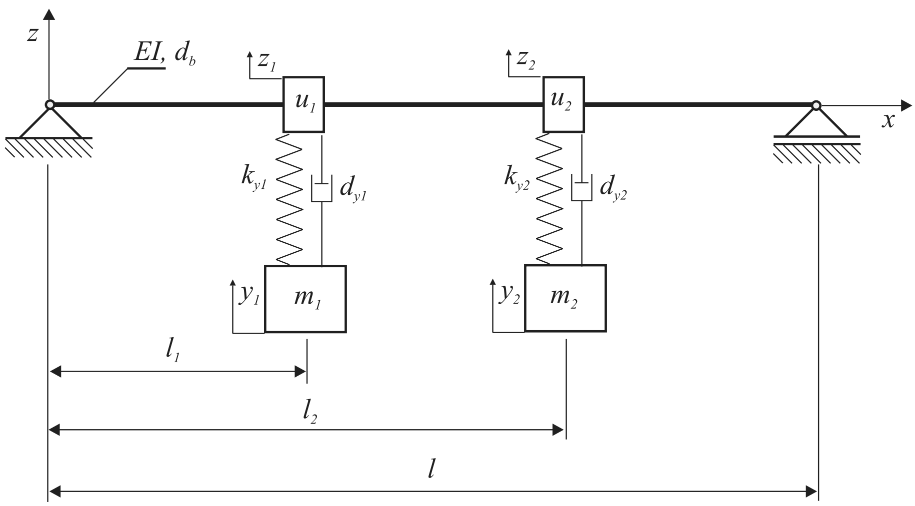

Fig. 1. Two van der Pol oscillators suspended on the elastic beam.

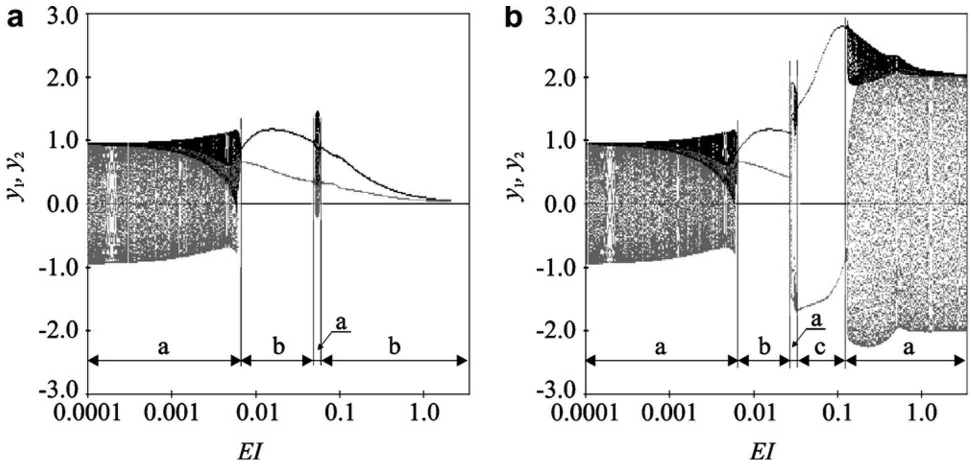

Fig. 2. Bifurcation diagram of the system (2) $l = 1.0$, $l_1 = 0.333$, $l_2 = 0.666$, $\vartheta = 0.2$, $u_1 = u_2 = 1.0$, $m_1 = m_2 = 1.0$, $d_{y1} = d_{y2} = 0.5$, $k_{y1} = 2.0$, $k_{y2} = 1.0$: (a) beam stiffness EI increases from 1.0e−4 to 3.9 and (b) EI decreases from 1.0e−4 to 3.9.

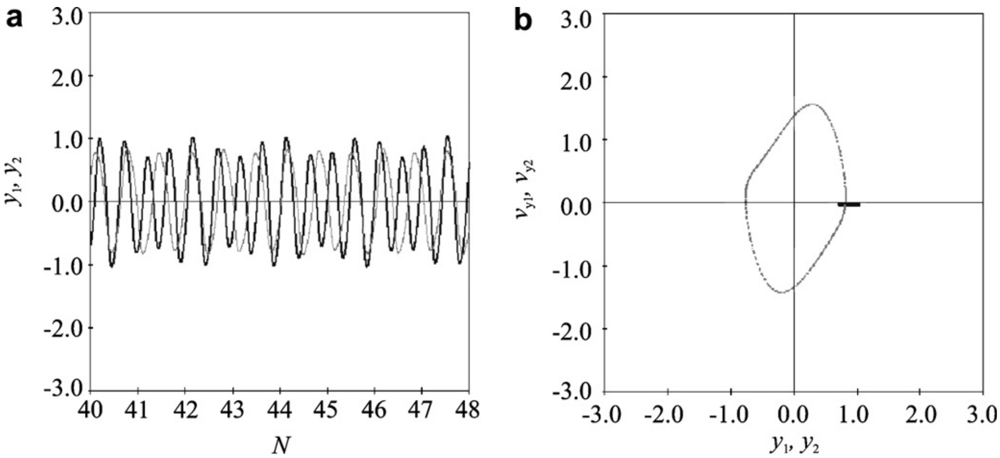

Fig. 3. Evolution of the system (2) $l = 1.0$, $l_1 = 0.333$, $l_2 = 0.666$, $\vartheta = 0.2$, $u_1 = u_2 = 1.0$, $m_1 = m_2 = 1.0$, $d_{yl} = d_{y2} = 0.5$, $k_{yl} = 2.0$, $k_{y2} = 1.0$, $EI = 0.002$: (a) time series and (b) Poincare map.

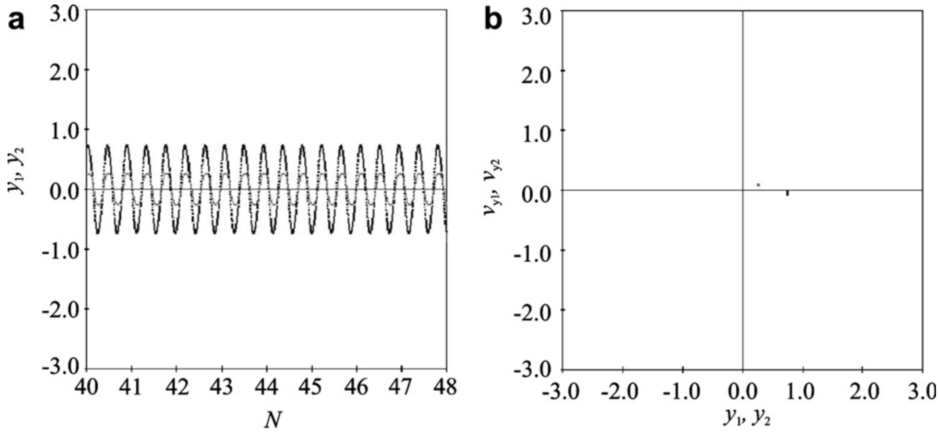

Fig. 4. Evolution of the system (2) $l = 1.0$, $l_1 = 0.333$, $l_2 = 0.666$, $\vartheta = 0.2$, $u_1 = u_2 = 1.0$, $m_1 = m_2 = 1.0$, $d_{y1} = d_{y2} = 0.5$, $k_{y1} = 2.0$, $k_{y2} = 1.0$, $EI = 0.1$: (a) time series and (b) Poincare map (as in Fig. 2a).

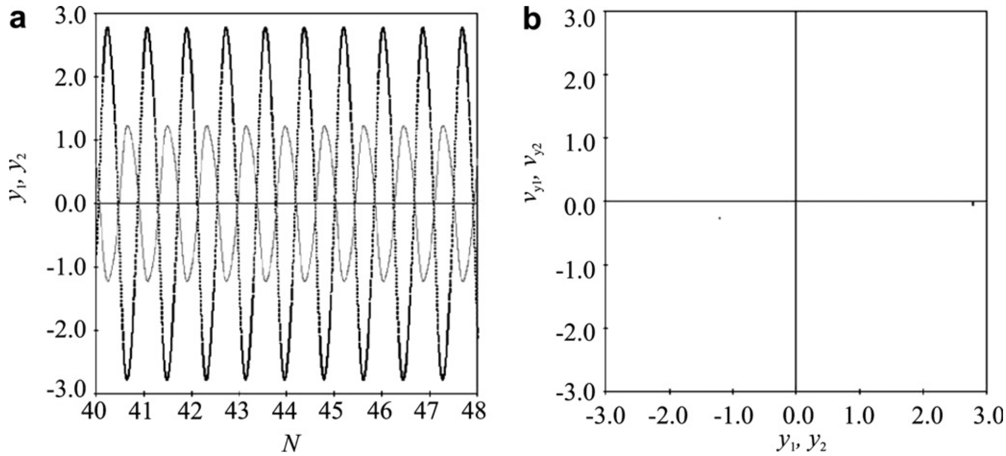

Fig. 5. Evolution of the system (2) $l = 1.0$, $l_1 = 0.333$, $l_2 = 0.666$, $\vartheta = 0.2$, $u_1 = u_2 = 1.0$, $m_1 = m_2 = 1.0$, $d_{y1} = d_{y2} = 0.5$, $k_{y1} = 2.0$, $k_{y2} = 1.0$, $EI = 0.1$: (a) time series and (b) Poincare map (as in Fig. 2b).

and b presents bifurcation diagrams of y_1 (grey marker)and y_2 (black marker) versus beam stiffness EI (in logarithmic scale). We have shown the position of the left oscillator y_1 in time when its velocity changes sign from positive to negative. The position of the right oscillator y_2 is shown in the same time. The same idea was applied to plots in Figs. 3–5. In Fig. 2a the stiffness of the beam EI, was assumed to increase from 1.0e−4 to 3.9. Initial conditions for minimal value of EI were taken as $y_{10} = y_{20} = 0.1$, $z_{10} = z_{20} = 0.0$, $v_{y10} = v_{y20} = v_{z10} = v_{z20} = 0.0$. Two kinds of motion character were observed:

(i) Displacements of van der Pol oscillators cause the oscillations of the beam with masses u_1 and u_2 located on it and disturb self-exited periodic evolution of the oscillators (initially with the frequencies α_{s01} and α_{s02}). In this case synchronization does not appear. Evolution of each oscillator is quasi-periodic (combination of self-excited oscillations with frequencies α_{s01} or α_{s02} and oscillations of the beam) as can be seen in Fig. 3a and b for $EI = 0.002$. Fig. 3a shows time series and Fig. 3b appropriate Poincare map.
(ii) For the given beam stiffness EI the synchronization of oscillators can be observed. System (2) exhibits periodic behavior with period 1. An example of this behavior is shown in Fig. 4a and b ($EI = 0.1$ has been taken). Fig. 4a presents time series and Poincare map is shown Fig. 4b.

Beam stiffness EI in the bifurcation diagram shown in Fig. 2b is in the same range as in Fig. 2a but now it decreases from 3.9 to 1.0e−4. Initial conditions for maximal value of EI were taken as $y_{10} = y_{20} = 0.1$, $z_{10} = z_{20} = 0.0$, $v_{y10} = v_{y20} = v_{z10} = v_{z20} = 0.0$. For large values of beam stiffness ($EI > 0.123$) the evolution of the system (2) is quasi-periodic (not a periodic one as in the case shown in Fig. 2a). In the range $0.059 < EI < 0.123$ oscillators behavior is periodic, synchronous, but has different form than the one observed in Fig. 2a. An example of this evolution observed for $EI = 0.1$ is shown in Fig. 5a and b as time series in Fig. 5a and a Poincare map in Fig. 5b. For lower values of beam stiffness EI, the system evolution is qualitatively the same as in Fig. 2a.

The results of our numerical investigations show that for $EI > 0.123$ one can observe coexistence of (at least) two attractors: periodic and quasi-periodic one. The same situation occurs in the range $0.059 < EI < 0.123$ where there are two different periodic attractors. In the second case independently on initial conditions the behavior of oscillators is periodic and synchronous but dependently on these conditions oscillators evolve in phase or anti-phase. The examples of initial conditions leading to both types of evolution are shown in Fig. 6. We assumed $EI = 0.1$, $z_{10} = z_{20} = 0.0$, $v_{y10} = v_{y20} = v_{z10} = v_{z20} = 0.0$ and the values of y_{10} and y_{20} were taken in the interval $[-1.0, 1.0]$.

To explain the synchronization mechanism, let us consider the evolution of van der Pol oscillator with kinematical excitation shown in Fig. 7. This case is very close to one when the oscillator is connected to elastic beam (as in Fig. 1). In the numerical analysis we assumed: $m = 1.0$, $d_y = 0.5$, $k_y = 1.0$. Our investigations have shown that this system can behave in two different ways. The examples for $Z = 0.5$ and two different values of ω ($\omega = 1.3$ and $\omega = 1.15$) are shown in Fig. 8a and b. It can exhibit quasi-periodic evolution, which is a combination of oscillations excited by kinematical signal with frequency ω and self-exited oscillations with frequency α_{s0} (the same as without kinematical excitation) as in Fig. 8a. The second possibility is a periodic evolution with excitation frequency ω (Fig. 8b) where the existence of kine-

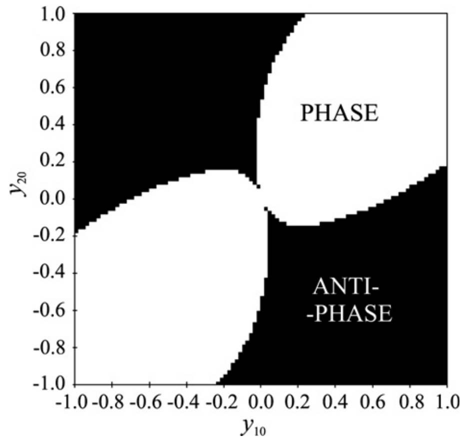

Fig. 6. Map of initial conditions leading to phase synchronization (white area) and antiphase synchronization (black area); $l = 1.0$, $l_1 = 0.333$, $l_2 = 0.666$, $\vartheta = 0.2$, $u_1 = u_2 = 1.0$, $m_1 = m_2 = 1.0$, $d_{y1} = d_{y2} = 0.5$, $k_{y1} = 2.0$, $k_{y2} = 1.0$, $EI = 0.1$, $z_{10} = z_{20} = 0.0$, $v_{y10} = v_{y20} = v_{z10} = v_{z20} = 0.0$.

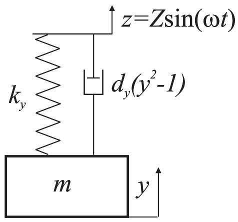

Fig. 7. Kinematically excited van der Pol oscillator.

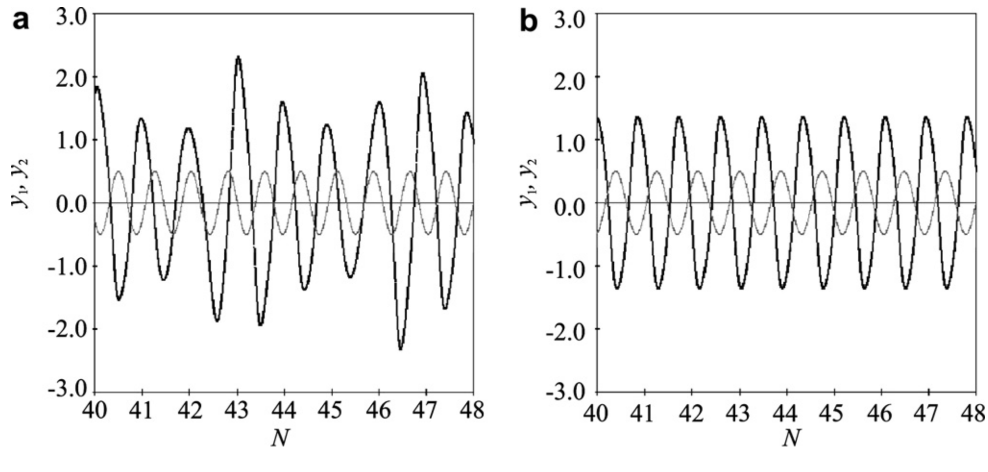

Fig. 8. Time series of kinematically excited van der Pol oscillator; $m = 1.0$, $d_y = 0.5$, $k_y = 1.0$, $Z = 0.5$: (a) $\omega = 1.3$ and (b) $\omega = 1.15$.

matical excitation cause the change of the frequency of self-exited oscillation period from α_{s0} to ω. The condition of such a modification of the frequency of self-exited oscillations (i.e., the oscillator is synchronized with kinematical excitation) is a large enough amplitude of kinematical excitation z and a sufficiently small difference between frequencies ω and α_{s0}. The minimum value of Z sufficient for synchronization is shown in Fig. 9.

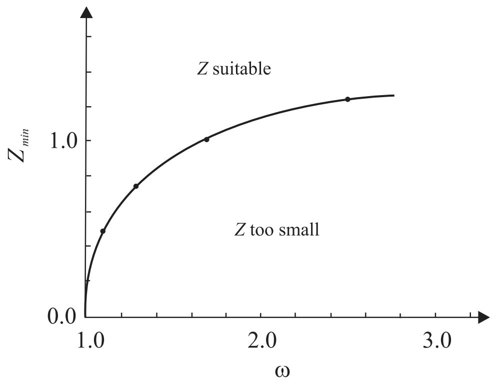

Fig. 9. Amplitudes kinematical excitation Z necessary for synchronization versus excitation frequency ω.

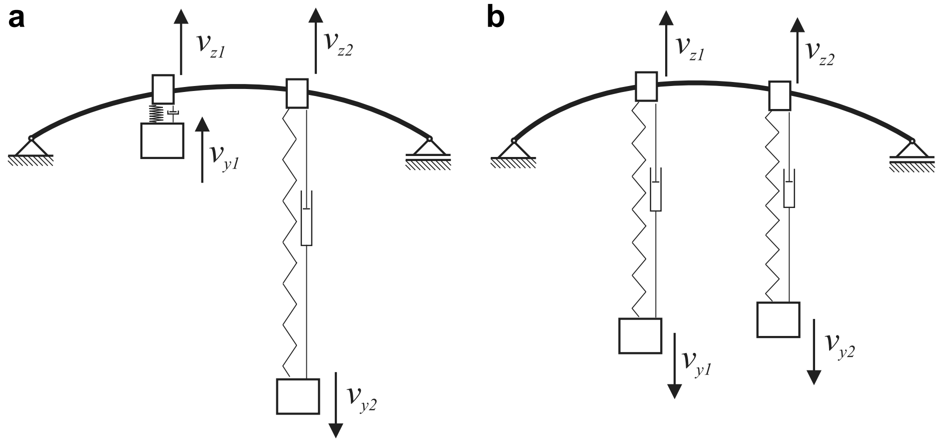

Fig. 10. Geometrical configurations of the beam-oscillators system: (a) eigenfrequency 1.218 and (b) eigenfrequency 2.328.

Our results showed that the displacement of the elastic beam with van der Pol oscillators connected to it can change frequency of self-exited oscillations and force both oscillators to adopt the same frequency which results in their synchronization. To answer the question what that common frequency will be let us come back to Figs. 4a and 5a which show time series of periodic oscillations of the beam with connected van der Pol oscillators for $EI = 0.1$ with different initial values. It is easy to see that frequencies of the periodic evolution are 2.31 in Fig. 4a and 1.21 in Fig. 5a. One can find that these frequencies are nearly equal to two of four eigenfrequencies of linear system (2) α_{1-4} (i.e., system without damping $\vartheta = d_{y1} = d_{y2} = 0$). These eigenfrequencies are respectively equal to 0.885, 1.218, 2.328 and 7.083. It was found also that the geometrical configuration of the beam–van der Pol oscillators system in these two cases is identical with the corresponding eigenvector of a linear system. The appropriate geometrical configurations are shown in Fig. 10a and b. Fig. 10a shows the case with eigenfrequency 1.218 and Fig. 10b one with eigenfrequency 2.328.

4. Conclusions

Our study showed that two van der Pol oscillators suspended on the elastic bean can synchronize when (i) frequencies of self-oscillations of both oscillators α_{s01} and α_{s02} are close to eigenfrequencies α_i of the linear system with four degree-of freedom, (ii) amplitudes of self-excited oscillations generated by van der Pol oscillators and amplitudes of oscillations of concentrated masses u_1 and u_2 are large enough to cause the change of self-excited frequencies α_{s01} and α_{s02} to one common frequency α_i, (iii) initial conditions allow the system to approach configuration which is close to appropriate eigenvector.

Summarizing, we have shown that van der Pol oscillators do not synchronize directly with each other. The oscillators synchronize their behavior with the evolution of the whole beam-oscillators system the configuration of which is determined by one of eigenvectors of its linear part. This synchronization mechanism explains the coexistence of different attractors as the phenomenon which occurs when the nonlinear system adopts its behavior (depending on the initial conditions) to different eigenfrequencies and eigenvectors of its linear part.

The similar synchronization phenomenon was observed in a large range of system parameters and for different locations of oscillators on the beam so it seems to be robust in the parameter space.

If one increases the number of oscillators, the system will be more dimensional and there will be more beam-oscillators geometrical configurations (like these in Fig. 10a and b) so it will be possible to observe a group (cluster) of oscillators which synchronize in phase while the rest of them synchronizes in antiphase. These results will be published elsewhere [20].

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